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CLASSICAL MECHANICS AND QUANTUM MECHANICS

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Abstract: Here I describe the difference between the classical mechanics and quantum mechanics. Also described the formulas involved in explanation of dynamical variables and their analogy between classical mechanics and quantum mechanics. Additionally, added the "operators" used for indication of the dynamical variables in Quantum mechanics.

Key words: Normalize- Phasor- embedded- effective phase angle-expectation value

1. Introduction:

As we know that classical mechanics explains the motion of the objects which moves with less than light velocity. They name as "Newtonian mechanics" and this can be applicable to all macroscopic bodies like planets to a small stone. Whereas the Quantum mechanics is called Relativistic mechanics applicable to bodies of less than Nano sized. In the view of classical mechanics, the motions of several of bodies can be easily explained by some fundamental formulas and they impart very accurate results.

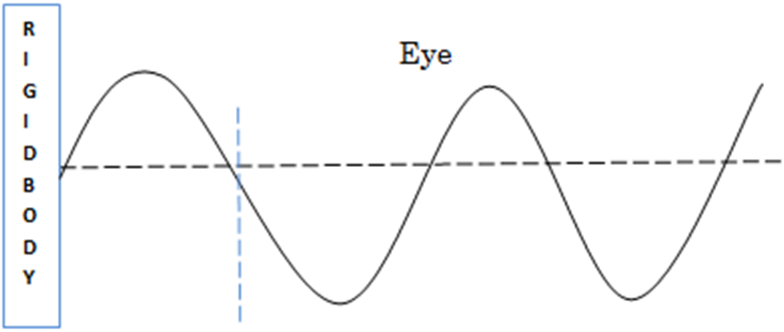
2. Classical mechanics says:

In macroscopic world, the system's static properties do not change with time. The mass of an object might be static property but the change in the system it explained by some dynamic variables. The way state of the system changes under particular actions is then explained by how the dynamical variable changes with time. Under these actions of forces some peculiar mathematical equations are derived to elaborate motion of the bodies. “Equation of motion” (most applicable equations)detailed the time dependent motion of the bodies. The parameter “time” more than zero imparts there some action on the macroscopic bodies.

For example take a system (a point mass) , mass “m” which exhibiting a static property. Let us consider the motion confines one dimensional linear space. According to classical mechanics, the state of the particle at any instant “t” is specified in terms of its position $x(t)$ and velocity $v_x(t)$. Some other dynamic properties such as linear momentum $p_x(t) = mv_x$, kinetic energy $T = \frac{mv^2x}{2}$, potential energy $V(x)$, tota energy $E = (T + V)$ etc, of this system depends only on “x” and v_x . The state of the system is known initially means that the numerical values of $x(0)$ and $v_x(0)$ are mentioned. The below given equation describes the action on the particle in terms of a force , F_x acting on the particle and this external force is proportional to the acceleration, $a_x = \frac{d^2x}{dt^2}$, where the mass (m)of the particle always remains constant.

$$F_x = ma_x = m\frac{d^2x}{dt^2} --- (1)$$

The sound waves are mechanical waves and some of beautiful examples are confirmed this hypothesis. Classical mechanics predicting the position and momentum of a particle simultaneously if initial position and the cause of force on the particle in the atom. But quantum mechanically it’s impossible and meaningless content by predicting the starting point of particle exactly in atom.



If a rope fixed at one end and the other is waved, then the observer on the other end will see a straight line and an ink mark on the rope observed by observer clearly and position of the mark can me identified accurately. Classically the equation takes place for the motion of that mark (we can imagine an electron in the place of ink mark),

$$y(t) = A \sin (wt + \emptyset) --- (2)$$

The displacement of wave with function time, t , is $y(t)$, "A" represents amplitude of the particle in wave and "w", and "Φ" are angular frequency and Phase of the particle. In addition we can precisely calculate the effective phase difference at point where other end of the rope fixed.

But if an electromagnetic wave falls on the surface of the rigid end, the reflected ray is superimposed with incident ray. A standing wave takes place but it does not observed by observer and very difficult to predict precisely the effective path difference at that moment.

The equation for this action is illustrate as $2\mu t \cos r \pm \pi = n\lambda --- (3)$

"μ" represents the refractive index of solid material, 't " , λ are the thickness of rigid body, and wavelength of light.

3. According to quantum mechanics the "state" of a system on atomic and subatomic scale is completely specified by a "state function".

The dynamics of the system is described by the time dependence of this state function. The state function is a function of set of selected variables, called "canonic variables" of the system.For example the case of a particle of mass "m" constrained to move in a linear space along x –axis. The state function, which is designated by the symbol Ψ , is a function of "x". The state of the particle changes with time is specified by Ψ(x,t) and the "wave function" of the particle also referred with the same. Sometimes the state function can also be expressed as a canonic conjugate variable to represent the position coordinate and linear momentum of the particle of the system. Mathematically it is shown as $\psi(P_x, t)$. The dynamic variables of the particle can be formulated in either equivalent form or in either representation form. If the dynamical variables use the form $\psi(x, t)$, it is said to be "Schrodinger representation" and $\psi(P_x, t)$ is used in "momentum representation".

The probabilistic, nature of the measurement process on the microscopic particles is imbedded in the physical interpretation of the state function. For example, the wave function $\psi(x, t)$ is in general complex function of *x* and *t* , meaning it is a phasor of the form $\psi = |\psi|e^{i\phi}$ with an amplitude $|\psi|$ and a phase Φ. The magnitude of the wave function , $|\psi(x, t)|$ gives statistical information on the results of measurement of the position of the particle. $|\psi(x, t)|^2 dx$ is then interpreted as the probability of finding a particle in the collection of particles. In quantum mechanics, the action on the dynamic system is generally specified by a "observable" property corresponding to the "potential energy operator", say $\hat{V}(r)$

In general, all dynamic properties are represented by "operators " that are functions of x and $\widehat{p_x}$. A "hat $\wedge$ " over a symbol in the language of quantum theory indicates that the symbol is mathematically an "operator", which in the Schrödinger representation can be a function of x and / or a differential operator involving x.

For example, the operator representing the linear momentum, $\widehat{p_x}$, in the Schrödinger representation is represented by an operator that is proportional to the first derivative with respect to x,

$$\widehat{p_x} = -i\hbar\frac{\partial}{\partial x} --- (4)$$

Where $\hbar$ is the plank's constant h divided by 2π. h is one of the fundamental constants and numerical value $h = 6.626 \times 10^{-27} erg - s$

Some of the operators listed below from the both classical & quantum mechanics,

Classical quantity	Quantum mechanical operator
Cartesian components of position x,y,z	$\hat{x}, \hat{y}, \hat{z}$
Position vector "r"	$\hat{r}$
Momentum "p"	$(-i\hbar\vec{\nabla})$
Cartesian components of linear momentum $p_x p_y p_z$	$\left(= -i\hbar\frac{\partial}{\partial x}, -i\hbar\frac{\partial}{\partial y}, = -i\hbar\frac{\partial}{\partial z}\right)$
Total energy E	$i\hbar\frac{\partial}{\partial t}$

The total energy of the system is generally represented as the "Hamiltonian " and usually represented by the symbol $\widehat{H}$. It is the sum of the kinetic energy and potential energy of the system as in Newtonian mechanics

$$\widehat{H} = \frac{\widehat{p}_x{}^2}{2m} + \widehat{V}(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \widehat{V}(x) ----(5)$$

The Hamiltonian, plays a crucial role in the equation of motion dealing with dynamics of quantum systems.

The main equation of motion is postulated by Schrodinger is that the time – rate of change of the state function is proportional to the Hamiltonian "operating" on the state function.

$$i\hbar\frac{\partial\psi}{\partial t} = \widehat{H}\psi --- (6)$$

For the 1- dimensional single particle system from Schrödinger representation gives a partial differential equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \widehat{V}(x)\right]\psi --- (7)$$

Equation (7) is basic equation of motion in quantum mechanics. Schrödinger's equation (6), in quantum mechanics is analogous to Newton's equation of motion, eq(1), in classical mechanics .

4. A fundamental distinction between classical mechanics and quantum mechanics is that, in classical mechanics, the state of the dynamic system is completely specified by position and velocity of each constituent part of the system. The accuracy of finding of a particle in a system is completely definite. But quantum mechanically it is impossible to give that accuracy because the velocity of the particle is so speed (equal to speed of light) at the same time its size is so small. Only probable prediction of position of particle can be decided. Boundary conditions on the position of particle will applied to range from x=0 to x =L, then the probability distribution function $|\psi(x)|^2$ is integrated over this range must be equal to 1and the wave function said to be normalize.

$$1 = \int_0^1 \Psi(x)^*\Psi(x)dx = \int_0^1 |\Psi(x)|^2 dx$$

If the wave function is normalized, the absolute value of the probability of finding the particle in the range from x to x+dx is $|\Psi(x)|^2 dx$. According, there is also an average value, $\langle x\rangle_\Psi$, of the position of the particle in the state Ψ , which is called the "expectation value" of the position of the particle.

$$\langle x\rangle_{\Psi} = \int_0^L \Psi^*(x)x\Psi(x)dx = \int_0^L x|\Psi(x)|^2 dx$$

5. **Conclusion:** The motion of a point object and its rest of dynamical properties can be easily determined by applying some fundamental physical phenomenon. But the exact position and momentum of a sub –atomic particles in atomic particles takes us towards challenging direction. Classically the prediction of the position and momentum of a sub atomic particles and forces supporting to these particles to move around in specific speed etc precisely explained. So difficulty in that accuracy and definiteness like size of the nucleus and non-existence of electrons in nucleus was clearly explained by Quantum mechanics. See the world in Quantum mechanical way reveals the secrets behind matter.

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